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Avoiding Higgs Fields

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We show that it is possible within the given structure of Yang–Mills gauge theories to construct massive models without Higgs fields. We also discuss briefly the fermion mass term in this approach.

Although the introduction of the Higgs fields and Higgs mechanism (Higgs, 1964*a*,*b*, 1966; Kibble, 1967) has been the only way to formulate massive gauge field theories, nevertheless it remains a phenomenological solution. In addition to the fundamental problems with Higgs fields, such as their existence, one has to deal with a lot of ad hoc parameters (for example, in the Higgs potential) which are responsible for spontaneous symmetry-breaking (SSB) channels. Furthermore, one has to choose a certain representation of the gauge group for the Higgs field to produce the desired SSB, as there exist different representations belonging to the same minimum of the Higgs potential but causing different SSB directions. There are various problems of the standard model that are related to the Higgs field, for example, the fermion mass relation. Altogether the situation of the Higgs field and the Higgs mechanism within the standard model seems similar to that of the epicycles and eccentrics in Ptolemaic astronomy, which were introduced to improve the (wrong) geocentric system (e.g., Sambursky, 1975).

More or less in the same manner as in Ptolemaic astronomy, one may also change the point of view of the given structure of gauge field theory to get a realistic ("massive") model.

We present here a possibility for massive Yang–Mills theories without Higgs fields and discuss briefly the problem of fermions in this approach.

To begin with, we mention a qualitative relation between our approach and that of the noncommutative geometry of gauge field theory, where one

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tries to understand conceptually the Higgs fields (Connes, 1989).[2] There one incorporates Higgs fields into some components of a generalized gauge potential, which is enlarged by virtue of extra degrees of freedom and a related symmetry structure. It is a well known fact that a new localized degree of freedom coresponds to a suitable potential. In our approach we need not enlarge the given structure of Yang–Mills theories. Here the scalar Higgs field is built up of the vacuum expectation value (VEV) of the nonvanishing pure gauge potential part of the usual gauge potential after a gauge transformation. This VEV can be considered as a classical background of gauge potentials which do not propagate, in contrast to the true protentials. However, the SSB concept used by us to produce pure gauge potentials is mathematically equivalent to extra degrees of freedom (Connes, 1989) which have to be reduced to the "usual" number of degrees of freedom.

Considering the Lagrange density for a $SU(n)$ Yang–Mills model

$$L \sim \mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}), \qquad F_{\mu\nu} \approx A_{\nu,\mu} - A_{\mu,\nu} + [A_\mu, A_\nu] \qquad \left({}_{,\nu} = \frac{\partial}{\partial x^\nu}\right) \tag{1}$$

let us decompose the gauge potentials in the above-mentioned manner:

$$A_\mu = g_{,\mu}g^{-1} + gA'_\mu g^{-1}; \qquad A_\mu = A^i_\mu T_i, \qquad [T_i, T_j] \sim \varepsilon_{ijk}T^K$$

$$A_\mu := B_\mu + \omega_\mu, \qquad F_{\mu\nu}(B_\mu) \equiv 0$$

$$F_{\mu\nu}(A_\mu) \sim F_{\mu\nu}(\omega_\mu) + O([B_\mu, \omega_\nu])$$

Here we suppress all factors and ω^i_μ are the usual gauge potentials, whereas B^i_μ are pure gauge potentials. $\langle B^i_\mu \rangle$ can be considered as a classical background of A^i_μ.[3] Furthermore, T_i, $i = 1, \ldots, n^2 - 1$, are generators of the $SU(n)$ group. This decomposition means that we consider the gauge potential A_μ in a special section of the fiber bundle or in a broken gauge where only some gauge transformations are allowed.

Usually pure gauge potentials are identified by

$$B_\mu \sim g_{,\mu}g^{-1}, \qquad F_{\mu\nu}(B_\mu) \equiv 0 \tag{2}$$

where $g \sim \exp(T_j\theta^j)$ is the matrix representation of a group element and θ^j are the group parameters.

[2]Indeed, there are other attempts to avoid the Higgs fields, but all of them work outside of the given structure of usual gauge theories. These models use either new scalar potentials or vector potentials or higher degrees of freedom. That means they only replace Higgs fields, but they cannot avoid them.

[3]This decomposition is analogous to that in general relativity, where one has $g_{\mu\nu}(x_\alpha) = \eta_{\mu\nu} + h_{\mu\nu}(x_\alpha)$. Here $\eta_{\mu\nu}$ is the constant Minkowski metric as classical background and $h_{\mu\nu}$ is effectively the real gravitational potential which propagates as gravitational waves.

These potentials can be transformed away by a suitable gauge transformation of the gauge potentials, which is only a change of section of the related fiber bundle:

$$A'_\mu = g^{-1} A_\mu g - g^{-1} g_{,\mu} \tag{3}$$

However, if the gauge symmetry is broken such that some gauge bosons (potentials) acquire mass, one cannot apply the whole group of gauge transformations on A_μ or B_μ. Thus, one cannot transform B_μ away although it is a pure gauge potential. In other words, SSB may be considered as the possibility of a reduced gauge transformation of gauge potentials and fermions. This reduction may be represented by the nonvanishing VEV of the Higgs fields or by the reduction of the allowed gauge transformation transforming $\langle B_\mu \rangle$ away. Let us consider such a (SSB), with $G = SU(n)$ and $F = SU(n)/SU(m)$ as broken subgroup of G,[4]

$$\begin{gathered} G \to H; \qquad H = SU(m) \\ g \in G, \qquad h \in H, \qquad f \in F, \qquad g\colon\ h \cdot f \\ h \sim \exp(T_l \theta^l), \qquad f \sim \exp(T_p \theta^p) \\ l = 1, \ldots, m^2 - 1; \qquad p = 1, \ldots, n^2 - m^2 - 1 \end{gathered} \tag{4}$$

Thus, after SSB we can perform gauge transformations like (3) only with the group elements of H, i.e.,

$$B'_\mu \sim h^{-1} B_\mu h - h^{-1} h_{,\mu} \tag{5}$$

where we suppress all factors. Inserting B_μ in (5),

$$B'_\mu \sim h^{-1}(g_{,\mu} g^{-1}) h - h^{-1} h_{,\mu}$$

we obtain

$$B'_\mu \sim f_{,\mu} f^{-1} \tag{6}$$

which is not zero, although $F_{\mu\nu}(B'_\mu) \equiv 0$.

Now the SSB takes place in the following manner: First, we suppose that some components of

$$\langle B^p_\mu \rangle \neq 0; \qquad p = 1, \ldots, n^2 - m^2 - 1$$

[4]The simplest case, where one can apply directly the SSB, is the case with $G = SU(M) \times SU(L)$, $H = SU(L)$, and $F = SU(M)$. We give here only a qualitative approximation of SSB for the more general case $G = SU(n)$. In the case of the adjoint representation for SSB vacuum one has of course $SU(m) \times U(1)$ and so on, but with respect to a qualitative analysis it is not important with or without $U(1)$.

This means that we originally have (because of $\langle B^j_\mu \rangle = 0$)

$$\langle B^j_\mu \rangle [T_j, T_i] \equiv 0; \qquad i, j = 1, \ldots, n^2 - 1$$

Now, assuming $\langle B^p_\mu \rangle \sim \langle (f_{,\mu} f^{-1})^p \rangle \neq 0$, we obtain for some generators T_l,

$$\langle B^p_\mu \rangle [T_l, T_p] = 0, \qquad l = 1, \ldots, m^2 - 1 \tag{7}$$

However, in view of $[T_p, T_q] \sim \varepsilon_{pqr} T^r$, we get the following relation for other generators:

$$\langle B^p_\mu \rangle [T_p, T_q] \neq 0 \tag{8}$$

with $\{T_p, T_q, T_r\} \in \mathscr{A}_F$.

The main question here is: Can we take some component of $\langle B_\mu \rangle \neq 0$ without disturbing the invariance properties of the vacuum? Indeed it is possible to do so, because B_μ is not a covariant quantity and has no observable meaning, as it is subjected to the gauge transformations. The only covariant quantity made up of gauge potentials which has to have an invariant vacuum is $F_{\mu\nu}$. In fact, $\langle F_{\mu\nu}(B_\mu) \rangle \equiv 0$ by definition. Thus, even if we take $\langle B_\mu \rangle \neq 0$, the relevant quantity has a zero ground state and it is invariant under all relevant transformations as the vacuum state.

It is not hard to see that the decomposed Lagrangian (1) contains mass terms for quantized potentials

$$\begin{aligned} F_{\mu\nu}(A_\mu) &\sim F_{\mu\nu}(\omega_\mu) + ([B_\mu, \omega_\nu] + [\omega_\mu, B_\nu]) \\ F^i_{\mu\nu}(A^i_\mu) &\sim F^i_{\mu\nu}(\omega^i_\mu) + \varepsilon^i_{jk}(B^j_\mu \omega^k_\nu + \omega^j_\mu B^k_\nu) \end{aligned} \tag{9}$$

Thus

$$L \sim \mathrm{Tr}\{F_{\mu\nu}(\omega_\mu) F^{\mu\nu}(\omega^\mu)\} - \mathrm{Tr}(B_\mu B^\mu \omega^\nu \omega_\nu) + \cdots \tag{10}$$

The mass term in the component version of (1) is proportional to the second term of (10), i.e.,

$$\begin{aligned} \varepsilon_{ijk} \varepsilon_{ibc} \langle B^j_\mu B^\mu_b \rangle \omega^k_\nu \omega^\nu_c &= -(\delta_{jc}\delta_{kb} - \delta_{jb}\delta_{kc}) \langle B^j_\mu B^\mu_b \rangle \omega^k_\nu \omega^\nu_c \\ m^2 \omega^k_\nu \omega^\nu_k &\sim \langle B^j_\mu \rangle \langle B^\mu_j \rangle \end{aligned} \tag{10$'$}$$

whereas the first term in the rhs of (10′) plays only the role of an interaction term. As pointed out earlier, if we suppose for some of the B_μ components that $\langle B^j_\mu \rangle \neq 0$, then some of the ω^k_ν will get masses through the second term of (10) or (10′).

Obviously, this is equivalent to the breakdown of symmetry by an adjoint representation, whereas in the case of Higgs fields one has the freedom of using different representations of the gauge group. However, in this case one has to base the special representation of Higgs fields needed for the desired SSB, whereas in our approach the SSB representation $\langle B_\mu \rangle$ is given in a natural way. We would like to mention here that there exist models with noncommutative geometry which are subject to the same adjoint representation constraint as in our case (e.g., Dubois-Violette *et al.*, 1990, and references therein).

Furthermore, there is not a great difference in standard models with respect to gauge bosons if one replaces the vector representation with the adjoint representation of our approach.[5] The reason is simply that an $SU(n)$ group breaks through a Higgs field in its vector representation in $SU(n-1)$, whereas it breaks down to[6] $SU(n-1) \times U(1)$ if the Higgs field is an adjoint representation of $SU(n)$. Thus there is no difference for these two channels in the case of $SU(2)$ breaking and therefore also in the case of standard models of $SU(2) \times U(1)$ or $SU(5)$.[7] we will discuss the exact pattern of the SSB for $SU(2) \times U(1)$ with $\langle B_\mu \rangle$ in a forthcoming paper.

For fermion masses we briefly discuss here the simple case of the electron only. From the QED Langrangian given by

$$L \sim \bar{\psi}_e \gamma^\mu (\partial_\mu - A_\mu) \psi_e + \cdots, \qquad \partial_\mu := \frac{\partial}{\partial X^\mu}$$

we have

$$\bar{\psi}_e \gamma^\mu A_\mu \psi_e = \bar{\psi}_e \gamma^\mu \langle B_\mu \rangle \psi_e + \bar{\psi}_e \gamma^\mu \omega_\mu \psi_e \tag{11}$$

with A_μ or ω_μ as the proton field and $\langle B_\mu \rangle$ as its "pure gauge" part. We have suppressed all other factors. Using the γ_0 Dirac matrix [instead of the unit matrix for the $U(1)$ generator], which couples to the A_μ potential, one can construct an invariant term from $\bar{\psi}_e \gamma^\mu \langle B_\mu \rangle \psi_e$. We have to choose from $\gamma^0 \langle B_\mu \rangle$ only the zero component with nonvanishing vacuum expectation value, i.e.

$$\gamma^0 \langle B_\mu \rangle := m \delta^{0\mu} \langle B_\mu \rangle \gamma_0 \tag{12}$$

Then one gets the desired mass term $m\bar{\psi}_e \psi_e$ from the first term of (11).[8] However, one has to balance between different coupling constants, so that in a realistic case the VEV may be proportional to m/e or some related quantity.

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[5]In the Glashow–Salam–Weinberg standard model the SSB occurs through a vector representation of $SU(2)_L$, thereby three of four bosons related to $SU(2)_L \times U(1)$ acquire masses.
[6]There exists a second channel of SSB with $SU(m) \times S(n-m) \times I(1)$ as the unbroken subgroup.
[7]The $SU(5)$ Georgi–Glashow model uses an adjoint and a vector representation for (SSB) Higgs fields.
[8]One can consider, for example, γ_0 in the chiral representation.

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